

Muon magnetic dipole moment and Higgs mass in supersymmetric SU(5) models

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Abstract

We study the muon magnetic dipole moment and the Higgs mass in the framework of the supersymmetric SU(5) models. In this analysis, all the relevant parameters in the Lagrangian are taken to be free; in particular, assumption of the universal scalar mass is *not* adopted. Negative search for the Higgs boson at the LEP II experiment sets an important constraint on the supersymmetric contribution to the muon magnetic dipole moment a_μ (SUSY). It is shown that, for a fixed value of the lightest Higgs mass, the maximum possible value of a_μ (SUSY) becomes significantly larger in the general SU(5) case compared to the case of the universal scalar mass (i.e., the case of the so-called “CMSSM”). We also point out that, if we take relatively large value of the trilinear scalar couplings, the constraint from the Higgs mass is drastically relaxed. In this case, a_μ (SUSY) can be as large as $\sim 50 \times 10^{-10}$ even for small value of $\tan \beta$ (say, for $\tan \beta = 5$).

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Currently, supersymmetry (SUSY) is regarded as one of the most attractive candidates of the new physics beyond the standard model. Most importantly, the minimal supersymmetric standard model (MSSM) is not only consistent with experimental constraints but also is suggested from precision measurements; precise measurements of the electroweak parameters strongly prefer a light Higgs ($m_h \leq 205$ GeV [1]) and the MSSM naturally predicts such a light Higgs boson. In addition, it is well known that three gauge coupling constants meet at the scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV if the renormalization group equations based on the MSSM are used. Thus, to construct a viable model of the grand unified theory (GUT), it is natural to introduce superpartners of the standard-model particles to realize the gauge coupling unification. Furthermore,

in supersymmetric models, the naturalness problem is solved because of the cancellation of the quadratic divergences between bosonic and fermionic loops.

As well as these, the Brookhaven E821 experiment provided a new motivation of SUSY. In February 2001 the Brookhaven E821 experiment reported their result on the precise measurement of the muon magnetic dipole moment (MDM) [2]:

$$a_\mu(\text{E821}) = 11\,659\,202(14)(6) \times 10^{-10}. \quad (1)$$

Comparing this value with the standard-model prediction [3], we find $a_\mu(\text{E821}) - a_\mu(\text{SM}) = 43(16) \times 10^{-10}$, meaning that the E821 result is about 2.6σ away from the standard-model prediction. If we take this deviation seriously, some new physics beyond the standard model is needed to explain this anomaly. Among various models, the MSSM can provide significant extra contribution to the muon MDM [4].

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Of course, precise value of the SUSY contribution to the muon MDM $a_\mu(\text{SUSY})$ depends on soft SUSY breaking parameters which are model-dependent. Therefore, it is important to study the SUSY contribution to the muon MDM in various models to see if the E821 anomaly can be well explained without conflicting various experimental constraints. Indeed, after the announcement of the E821 result, there have been many works which discussed the SUSY contribution to the muon MDM in various cases [5]. In particular, with Komine and Yamaguchi, one of the authors (T.M.) pointed out that, in the unconstrained MSSM, $a_\mu(\text{SUSY})$ can be large enough to explain the deviation in wide parameter region.

Since the GUT is a strong motivation to introduce SUSY, it is reasonable to ask if the E821 anomaly can be explained even in the framework of SUSY GUTs. Once the unification of the gauge groups is assumed, some of the coupling constants and mass parameters should also obey the unification conditions, and hence the number of the free parameters is reduced compared to the case of the unconstrained MSSM. Thus, in SUSY GUTs, it is non-trivial whether the SUSY contribution to the muon MDM can become large enough in parameter region consistent with other experimental constraints. Previously, in several works, the SUSY contribution to the muon MDM is studied in SUSY SU(5) models. In those works, however, a very strong assumption is adopted, that is, the universal scalar mass at the unification scale. (Such a model is sometimes called “constrained MSSM” or “CMSSM”.) In general SUSY SU(5) GUTs, however, the universal scalar mass is not necessarily realized, and hence such an assumption imposes too strong constraints on the model. Indeed, there are models which do not predict the universal scalar mass. In addition, even if the universal scalar mass is somehow realized at the cutoff scale of the theory which naturally is the gravitational scale, the renormalization group effect spoils the universality. Therefore, it is necessary to study the muon MDM in a general framework of SUSY GUTs, and in this Letter we consider the SUSY contribution to the muon MDM in a general framework of the SUSY SU(5) models.

Before going into the details, we make several comments. First, in order to eliminate the model-dependence as much as possible, we do not consider

constraints from flavor and CP violating processes which are sensitive to small off-diagonal elements in the sfermion mass matrices. Such off-diagonal elements are hard to predict, and SUSY contributions to flavor and CP violations are significantly affected if the values of such off-diagonal elements are changed. Second, we do not take account of cosmological constraints. In particular, we do not require that the thermal relic of the lightest superparticle (LSP) be the cold dark matter (CDM). This is because the relic density of the LSP depends on cosmological scenarios. For example, if a late-time entropy production exists, the relic density of the LSP is changed [6]. Furthermore, the LSP is not the only particle-physics candidate of the CDM; for example, axion may be the CDM. As will be discussed, the model is still severely constrained by the lightest Higgs mass m_h even after excluding these constraints. In the following discussion, we will see that the SUSY contribution to the muon MDM is significantly constrained in some case once we impose the constraint on the lightest Higgs mass.

We begin our discussion by introducing the model we consider. We study SUSY SU(5) models. In this framework, the low-energy effective theory below the GUT scale M_{GUT} is the MSSM which contains the chiral superfields $Q_i(\mathbf{3}, \mathbf{2}, 1/6)$, $U_i^c(\mathbf{3}^*, \mathbf{1}, -2/3)$, $D_i^c(\mathbf{3}^*, \mathbf{1}, 1/3)$, $L_i(\mathbf{1}, \mathbf{2}, -1/2)$, $E_i^c(\mathbf{1}, \mathbf{1}, 1)$, $H_u(\mathbf{1}, \mathbf{2}, 1/2)$, and $H_d(\mathbf{1}, \mathbf{2}, -1/2)$ (where we denote the gauge quantum numbers of the SU(3)_C, SU(2)_L, and U(1)_Y gauge interactions in the parentheses) as well as vector superfields describing the SU(3)_C, SU(2)_L, and U(1)_Y gauge multiplets. Here, the subscript i is the flavor index which runs from 1 to 3. With these superfields, the relevant part of the superpotential is given by¹

$$W_{\text{MSSM}} = H_u U_i^c [Y_U]_{ij} Q_j + H_d D_i^c [Y_D]_{ij} Q_j + H_d E_i^c [Y_E]_{ij} L_j + \mu_H H_u H_d, \quad (2)$$

where Y_U , Y_D , and Y_E are Yukawa matrices for up-, down-, and electron-type fermions, respectively, and μ_H is the SUSY invariant Higgs mass. In addition, the

¹ For simplicity, we omit the gauge indices. Our sign convention for the μ_H parameter and the gaugino masses is the same as that used in Ref. [7].

soft SUSY breaking terms are

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -[m_{\tilde{Q}}^2]_{ij} \tilde{Q}_i^* \tilde{Q}_j - [m_{\tilde{U}^c}^2]_{ij} \tilde{U}_i^{c*} \tilde{U}_j^c \\ & - [m_{\tilde{D}^c}^2]_{ij} \tilde{D}_i^{c*} \tilde{D}_j^c - [m_{\tilde{L}}^2]_{ij} \tilde{L}_i^* \tilde{L}_j \\ & - [m_{\tilde{E}^c}^2]_{ij} \tilde{E}_i^{c*} \tilde{E}_j^c - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d \\ & - (H_u \tilde{U}_i^c [A_{\tilde{U}}]_{ij} \tilde{Q}_j - H_d \tilde{D}_i^c [A_{\tilde{D}}]_{ij} \tilde{Q}_j \\ & \quad - H_d \tilde{E}_i^c [A_{\tilde{E}}]_{ij} \tilde{L}_j + \text{h.c.}) \\ & - (B_\mu H_u H_d + \text{h.c.}) \\ & - \frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{G} \tilde{G} + \text{h.c.}). \quad (3)\end{aligned}$$

In our analysis, we impose the radiative electroweak symmetry breaking condition; we determine μ_H and B_μ parameters so that $v^2 \equiv \langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2 \simeq (174 \text{ GeV})^2$ and $\tan \beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$ are correctly obtained.

In the MSSM, all the soft SUSY breaking parameters given in Eq. (3) are free parameters. In the framework of the SU(5), however, that is not the case. Since \tilde{Q} , \tilde{U}^c , and \tilde{E}^c (\tilde{D}^c and \tilde{L}) are embedded in **10** (**5**) representation of SU(5), soft SUSY breaking parameters for these sfermions should be unified at the GUT scale. We parameterize the soft SUSY breaking parameters at the GUT scale as²

$$M_1 = M_2 = M_3 \equiv M_{1/2}, \quad (4)$$

$$[m_{\tilde{Q}}^2]_{ij} = [m_{\tilde{U}^c}^2]_{ij} = [m_{\tilde{E}^c}^2]_{ij} \equiv m_{\mathbf{10}}^2 \delta_{ij}, \quad (5)$$

$$[m_{\tilde{D}^c}^2]_{ij} = [m_{\tilde{L}}^2]_{ij} \equiv m_{\mathbf{5}}^2 \delta_{ij}, \quad (6)$$

$$m_{H_u}^2 \equiv m_{H5}^2, \quad m_{H_d}^2 \equiv m_{H\bar{5}}^2, \quad (7)$$

Notice that, in the most general approach, the soft SUSY breaking masses for the sfermions are not required to be proportional to δ_{ij} , and sizable off-diagonal elements in the sfermion mass matrices are possible. Such off-diagonal elements are, however, severely constrained since they induce various flavor (and CP) violating processes like $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, and $B^0 - \bar{B}^0$ mixings, $b \rightarrow s\gamma$, $\mu \rightarrow e\gamma$, and so

² We assume a simple unification condition for the down-type and electron-type Yukawa matrices although it does not reproduce realistic fermion mass texture for the first and second generation fermions. Our following discussions are, however, insensitive to the parameter $A_{\tilde{D}}$, and hence this assumption does not change our results significantly.

on [8]. In addition, in our following analysis, we focus on the muon MDM and the lightest Higgs mass which are insensitive to the flavor violations in the sfermion mass matrices. Thus, we neglect the effect of the off-diagonal elements in the following discussions. In summary, we parameterize the soft SUSY breaking parameters at the electroweak scale using the following parameters:

$$M_{1/2}, \quad m_{\mathbf{10}}, \quad m_{\mathbf{5}}, \quad m_{H5}, \quad m_{H\bar{5}}, \\ a_{\tilde{U}}, \quad a_{\tilde{E}}, \quad \tan \beta, \quad \text{sign}(M_{1/2} \mu_H). \quad (8)$$

Once these parameters are given, we can calculate the muon MDM and the lightest Higgs mass as well as the mass spectrum of the superparticles.

Let us next consider how the muon MDM and the lightest Higgs mass behave in this framework. In the MSSM, the supersymmetric contribution to the muon MDM is from chargino–sneutrino and neutralino–smuon loop diagrams. The most important point is that $a_\mu(\text{SUSY})$ is enhanced when $\tan \beta$ is large. In the limit $\tan \beta \gg 1$, the SUSY contribution to the muon MDM is approximately given by [4]

$$\begin{aligned}a_\mu(\text{SUSY}) \simeq & g_1^2 m_\mu^2 M_1 \mu_H \tan \beta \\ & \times \left[I_5^1(M_1^2, M_1^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}R}^2, m_{\tilde{\mu}R}^2) \right. \\ & \quad + I_5^1(M_1^2, M_1^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}R}^2) \\ & \quad - I_5^1(M_1^2, M_1^2, \mu_H^2, m_{\tilde{\mu}R}^2, m_{\tilde{\mu}R}^2) \\ & \quad - I_5^1(M_1^2, \mu_H^2, \mu_H^2, m_{\tilde{\mu}R}^2, m_{\tilde{\mu}R}^2) \\ & \quad + \frac{1}{2} I_5^1(M_1^2, M_1^2, \mu_H^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}L}^2) \\ & \quad \left. + \frac{1}{2} I_5^1(M_1^2, \mu_H^2, \mu_H^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}L}^2) \right] \\ & + g_2^2 m_\mu^2 M_2 \mu_H \tan \beta \\ & \times \left[-\frac{1}{2} I_5^1(M_2^2, M_2^2, \mu_H^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}L}^2) \right. \\ & \quad - \frac{1}{2} I_5^1(M_2^2, \mu_H^2, \mu_H^2, m_{\tilde{\mu}L}^2, m_{\tilde{\mu}L}^2) \\ & \quad + 2 I_4^0(M_2^2, M_2^2, \mu_H^2, m_{\tilde{\nu}}^2) \\ & \quad - I_5^1(M_2^2, M_2^2, \mu_H^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2) \\ & \quad + 2 I_4^0(M_2^2, \mu_H^2, \mu_H^2, m_{\tilde{\nu}}^2) \\ & \quad \left. - I_5^1(M_2^2, \mu_H^2, \mu_H^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2) \right], \quad (9)\end{aligned}$$

where

$$I_q^p(m_1^2, \dots, m_q^2) \equiv \int \frac{d^4k}{(2\pi)^4 i} \frac{(k^2)^p}{(k^2 - m_1^2) \dots (k^2 - m_q^2)}, \quad (10)$$

and $m_{\tilde{\mu}L}^2 \equiv [m_L^2]_{22}$, $m_{\tilde{\nu}L}^2 \equiv [m_L^2]_{22}$, and $m_{\tilde{\mu}R}^2 \equiv [m_{\tilde{E}^c}^2]_{22}$. For example, taking $m_{\tilde{\mu}L}^2 = m_{\tilde{\mu}R}^2 = M_2^2 = \mu_H^2 \equiv m_{\text{SUSY}}^2$, and neglecting the $U(1)_Y$ contribution, $a_\mu(\text{SUSY})$ becomes

$$a_\mu(\text{SUSY}) \simeq \frac{5g_2^2}{192\pi^2} \frac{m_\mu^2}{m_{\text{SUSY}}^2} \text{sign}(M_2\mu_H) \tan\beta. \quad (11)$$

When $\tan\beta$ is large, $a_\mu(\text{SUSY})$ can be sizable even if the superparticles are heavy. From Eq. (11), however, one easily sees that, when $\tan\beta$ is small, (some of) the superparticles are required to be light so that $a_\mu(\text{SUSY})$ becomes large enough to explain the BNL E821 anomaly. In addition, it is also important to note that $a_\mu(\text{SUSY})$ is proportional to $\text{sign}(M_2\mu_H)$ in the large $\tan\beta$ limit. Motivated by the E821 anomaly, hereafter, we take $\text{sign}(M_2\mu_H)$ to be positive.

For the case of light superparticles, we must consider various experimental constraints. First of all, negative searches for the superparticles set lower bounds on the masses of the superparticles. In this Letter, as a guideline, we require that all the charged superparticles be heavier than 100 GeV [9].³

In addition, lower bound on the Higgs mass derived by the LEP II experiment [10],

$$m_h \geq 113.5 \text{ GeV}, \quad (12)$$

provides a severe constraint when $\tan\beta$ is small. To understand this fact, it is instructive to see the leading-log formula for the lightest Higgs mass in the MSSM in the decoupling limit [11]:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \log \frac{m_t^2}{m_{\tilde{t}}^2}, \quad (13)$$

where m_Z is the Z -boson mass, $m_{\tilde{t}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ is the geometric mean of the two stop mass eigenvalues $m_{\tilde{t}_1}$

and $m_{\tilde{t}_2}$, and m_t is the top quark mass. (Hereafter, we use $m_t = 174.3 \text{ GeV}$ [9] unless otherwise mentioned.) Here, the first term is the tree-level contribution which becomes larger when $\tan\beta$ is large. On the contrary, the second term is the radiative correction from the top-stop loops, and is enhanced when the stops become heavier. Thus, when $\tan\beta$ is small, the stop masses are required to be heavy to satisfy the constraint (12).

At this point, it is natural to wonder if the two requirements, one from the muon MDM and the other from the Higgs mass, can be simultaneously satisfied when $\tan\beta$ is not large. To answer this question, it is crucial to study the mass spectrum and mixings of the superparticles at the electroweak scale.

In order for precise calculations of physical quantities as functions of the fundamental parameters listed in (8), we first calculate the MSSM parameters at the SUSY scale μ_{SUSY} . For this purpose, we use the renormalization group equations based on the standard model for the scale $\mu < \mu_{\text{SUSY}}$, and those based on the MSSM for $\mu_{\text{SUSY}} < \mu < M_{\text{GUT}}$. The parameter μ_{SUSY} should be regarded as a typical mass scale of the superparticles; in the following analysis, we take μ_{SUSY} to be the geometric mean of the stop masses unless otherwise mentioned. Then, using the parameters at $\mu = \mu_{\text{SUSY}}$, we calculate the mass spectrum and mixings of the superparticles as well as other physical quantities. Using the formula given in [4], we calculate the SUSY contribution to the muon MDM. In addition, we also calculate the lightest Higgs boson mass taking account of the dominant two-loop radiative corrections using **FeynHiggsFast** package [12].

Before going into the discussion about $a_\mu(\text{SUSY})$ and m_h , we first study the behavior of the sfermion masses (in particular, stop and smuon masses). Although the boundary conditions for the soft SUSY breaking parameters are given in Eqs. (4)–(7), the soft SUSY breaking parameters at the electroweak scale change because of the renormalization group effects. Taking $\mu_{\text{SUSY}} = 500 \text{ GeV}$ and $\tan\beta = 5$, we obtain

$$m_{\tilde{\mu}L}^2(\mu = \mu_{\text{SUSY}}) \simeq m_{\tilde{5}}^2 + 0.03m_{H5}^2 - 0.03m_{H\bar{5}}^2 + 0.49M_{1/2}^2, \quad (14)$$

$$m_{\tilde{\mu}R}^2(\mu = \mu_{\text{SUSY}}) \simeq m_{\tilde{10}}^2 - 0.07m_{H5}^2 + 0.07m_{H\bar{5}}^2 + 0.15M_{1/2}^2, \quad (15)$$

³ In the parameter region we are interested in, the first and second generation squarks as well as gluino are heavier than 300 GeV, and the experimental constraints on their masses are satisfied. Thus, we require $m_{\tilde{t}_1} > 100 \text{ GeV}$ [9].

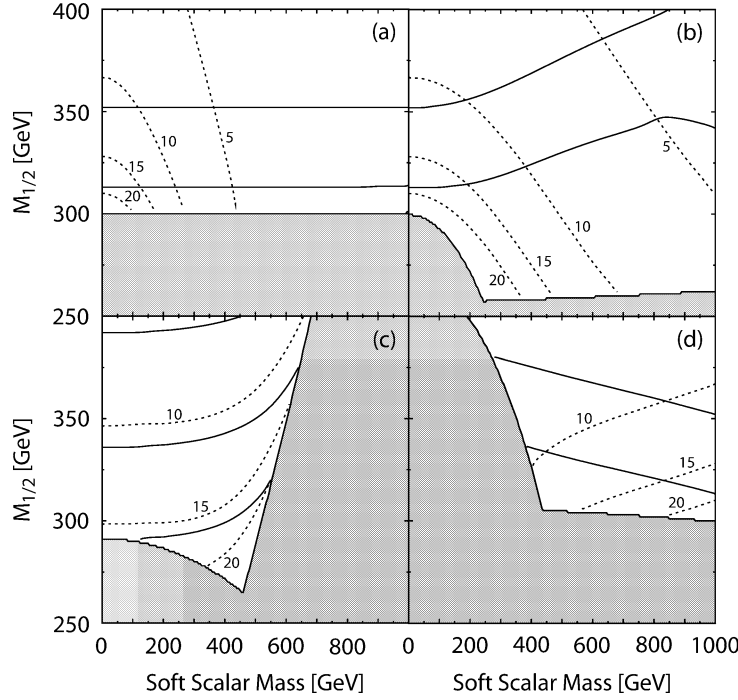


Fig. 1. Contours of constant $a_\mu(\text{SUSY})$ (dotted). (Values of $a_\mu(\text{SUSY})$ are shown in the figures in units of 10^{-10} .) The vertical axis is $M_{1/2}$, and the horizontal axis is (a) m_5 , (b) m_{10} , (c) m_{H5} , and (d) m_{H5} . Here, we take $\tan\beta = 5$, and $a_{\tilde{U}} = a_{\tilde{E}} = 0$. In addition, other parameters are (a) $m_{10} = 0$, $m_{H5} = 520$ GeV, $m_{H\bar{5}} = 1$ TeV, (b) $m_5 = 0$, $m_{H5} = 520$ GeV, $m_{H\bar{5}} = 1$ TeV, (c) $m_5 = 0$, $m_{10} = 0$, $m_{H\bar{5}} = 1$ TeV, and (d) $m_5 = 0$, $m_{10} = 0$, $m_{H5} = 520$ GeV. The shaded region is excluded by the negative searches for charged superparticles. Contours of the constant m_h are also shown in the solid lines ($m_h = 110$ GeV, 111 GeV (and 112 GeV for (c)) from below).

$$m_{\tilde{t}_L}^2 (\mu = \mu_{\text{SUSY}}) \simeq 0.75m_{10}^2 - 0.13m_{H5}^2 + 0.01m_{H\bar{5}}^2 + 4.12M_{1/2}^2 - 0.11a_{\tilde{U}}M_{1/2} - 0.03a_{\tilde{U}}^2, \quad (16)$$

$$m_{\tilde{t}_R}^2 (\mu = \mu_{\text{SUSY}}) \simeq 0.51m_{10}^2 - 0.20m_{H5}^2 - 0.04m_{H\bar{5}}^2 + 2.94M_{1/2}^2 - 0.23a_{\tilde{U}}M_{1/2} - 0.06a_{\tilde{U}}^2, \quad (17)$$

where $m_{\tilde{t}_L}^2 \equiv [m_{\tilde{Q}}^2]_{33}$, and $m_{\tilde{t}_R}^2 \equiv [m_{\tilde{U}^c}^2]_{33}$. From these relations, we expect rich particle mass spectrum. This model has a significant contrast with the CMSSM where the universal scalar mass is assumed: $m_{10}^2 = m_5^2 = m_{H5}^2 = m_{H\bar{5}}^2 \equiv m_0^2$. In the CMSSM, all the sfermion masses have strong correlations because all the sfermion masses are increased (decreased) if we adopt larger (smaller) values of m_0 and/or $M_{1/2}$. Thus, in the CMSSM, it is difficult to explain the E821 anomaly without conflicting the Higgs mass constraint if $\tan\beta$ is not large. In the general SUSY

SU(5) model, however, this is not the case since the correlation among the sfermion masses becomes weak. This fact has a very important implication as we will see below.

Now, we are at the position to discuss the SUSY contribution to the muon MDM as well as the lightest Higgs mass. In fact, m_h is sensitive to the value of $a_{\tilde{U}}$. Therefore, we split our discussion into two cases: one with relatively small $a_{\tilde{U}}$ and the other with large $a_{\tilde{U}}$.

We start our discussion with the former case. In Fig. 1, we plot contours of the constant $a_\mu(\text{SUSY})$ and m_h on m_5 vs. $M_{1/2}$, m_{10} vs. $M_{1/2}$, m_{H5} vs. $M_{1/2}$, and $m_{H\bar{5}}$ vs. $M_{1/2}$ planes, with $a_{\tilde{U}} = 0$. Notice that, in the parameter region we discuss below, we checked that the LSP is the neutral superparticles (the lightest neutralino or the sneutrino).

First, we discuss behaviors of $a_\mu(\text{SUSY})$. For this purpose, let us point out that the dominant contri-

bution to $a_\mu(\text{SUSY})$ is from the chargino–sneutrino diagram. Consequently, $a_\mu(\text{SUSY})$ is more enhanced with lighter charginos and lighter left-handed sleptons. Based on this fact, dependence on $M_{1/2}$ can be understood; for larger value of $M_{1/2}$, heavier superparticles are realized, resulting in suppressed $a_\mu(\text{SUSY})$. In addition, $m_{\tilde{5}}$ dependence is also trivial; with larger value of $m_{\tilde{5}}$, the left-handed slepton masses become larger and hence $a_\mu(\text{SUSY})$ becomes smaller (see Fig. 1(a)). Slight dependence on $m_{H\tilde{5}}$ is from the renormalization group effect on $m_{\tilde{\mu}_L}$. As can be seen in Eq. (14), $m_{H\tilde{5}}$ gives a negative contribution to $m_{\tilde{\mu}_L}^2$. Thus, to obtain a larger value of $a_\mu(\text{SUSY})$, $m_{H\tilde{5}}$ should be increased (see Fig. 1(d)). Dependences on m_{10} and m_{H5} arise since the μ_H parameter determined by the radiative electroweak symmetry breaking condition depends on these parameters; μ_H increases for larger value of m_{10} and for smaller value of m_{H5} . Since μ_H (almost) corresponds to the Higgsino-like chargino mass, larger value of μ_H gives rise to smaller value of $a_\mu(\text{SUSY})$. This results in the behaviors of $a_\mu(\text{SUSY})$ shown in Figs. 1(b) and 1(c).

From Fig. 1, we see that, in order to have a large SUSY contribution to the muon MDM, $m_{\tilde{5}}$ and m_{10} are preferred to be small while m_{H5} and $m_{H\tilde{5}}$ should be sizable. It is notable that such a situation may be compatible even with the SO(10) unification models where $m_{\tilde{5}} = m_{10}$ holds.⁴ Indeed, we numerically checked that, for $a_{\tilde{\nu}} = 0$, the maximum possible values of $a_\mu(\text{SUSY})$ given below are almost unchanged even if we impose the relation $m_{\tilde{5}} = m_{10}$. Of course, in a simple SO(10) model, $m_{H5} = m_{H\tilde{5}}$ since H_u and H_d are in the same **10** representation of SO(10). In this case, $a_\mu(\text{SUSY})$ is suppressed. However, H_u and H_d may originate from different SO(10) multiplets and in this case, m_{H5} and $m_{H\tilde{5}}$ become independent. For example, such a non-trivial Higgs sector may be required to realize the observed structure of the Yukawa matrices.

Now, let us consider the Higgs mass. The Higgs mass is sensitive to $M_{1/2}$. This is because, in the

parameter region given in the figures, stop masses are primarily determined by $M_{1/2}$. Importantly, as $M_{1/2}$ increases, the stop masses are more enhanced, resulting in larger value of m_h . On the contrary, the Higgs mass is relatively insensitive to the scalar masses.

It is interesting to plot the maximum possible value of $a_\mu(\text{SUSY})$ as a function of the lightest Higgs mass. For this purpose, we vary the parameters $m_{\tilde{5}}$, m_{10} , m_{H5} , $m_{H\tilde{5}}$, and $M_{1/2}$ from 0 to m_{max} , where we take m_{max} to be 500 GeV, 1 TeV, and 2 TeV, and obtain the upper bound on $a_\mu(\text{SUSY})$ for a given value of m_h . The results are shown in Fig. 2. As m_h increases, the maximum possible value of $a_\mu(\text{SUSY})$ decreases. This is because, to obtain a larger value of m_h , $M_{1/2}$ is required to be large to enhance the radiative correction by pushing up the stop masses through the running effects. As a result, other sparticle masses are also suppressed for larger value of m_h and the upper bound becomes smaller. In addition, we can find a “kink” on each plot. This is from the fact that, to obtain the maximum possible value of $a_\mu(\text{SUSY})$, $m_{H\tilde{5}}$ is preferred to be large. However, when $M_{1/2}$ is small, the stau becomes lighter than the experimental bound if $m_{H\tilde{5}}$ is too large. Therefore, as m_h is reduced, the upper bound on $m_{H\tilde{5}}$ becomes smaller than m_{max} . On the contrary, when m_h is large enough, $m_{H\tilde{5}} = m_{\text{max}}$ is allowed. The kink corresponds to the boundary of these two parameter regions. If we change m_{max} , the upper bound on $a_\mu(\text{SUSY})$ also changes; when m_h is large, the maximum possible value of $a_\mu(\text{SUSY})$ increases by adopting larger value of m_{max} .

From Fig. 2 we see that the negative search for the Higgs boson at LEP II places a severe constraint on the possible value of the SUSY contribution to the muon MDM for small $\tan\beta$ case. For $\tan\beta \lesssim 5$, $a_\mu(\text{SUSY})$ cannot explain the E821 anomaly even at the 2- σ level if we adopt $m_{\text{max}} = 1$ TeV. Of course, with larger value of $\tan\beta$, $a_\mu(\text{SUSY})$ may become larger and it is possible to explain the E821 anomaly.

We should note here that the result depends on the top quark mass, since the radiative correction to the lightest Higgs mass is sensitive to m_t . The lightest Higgs mass is enhanced for larger value of m_t . Therefore, for a given value of m_h , we can push up the max-

⁴ If the D -term contribution is sizable, this relation does not hold. In the following, we neglect the D -term contribution to the soft scalar masses.

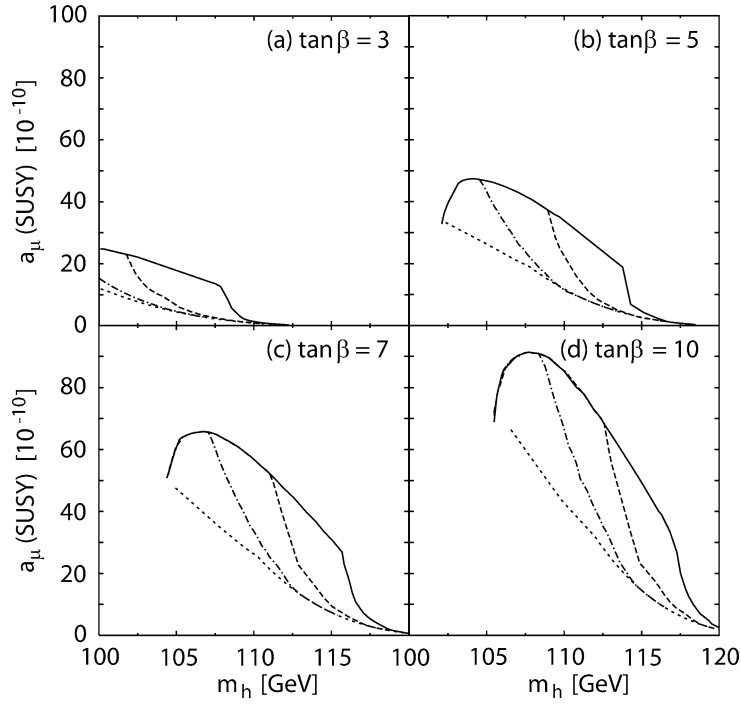


Fig. 2. Maximum possible value of $a_\mu(\text{SUSY})$ in units of 10^{-10} as a function of the lightest Higgs mass m_h . We take $m_{\text{max}} = 500$ GeV (dot-dashed), 1 TeV (dashed), and 2 TeV (solid), $a_{\tilde{U}} = a_{\tilde{E}} = 0$, and (a) $\tan \beta = 3$, (b) $\tan \beta = 5$, (c) $\tan \beta = 7$, and (d) $\tan \beta = 10$. The case with the CMSSM is also shown in the dotted lines.

imum possible value of $a_\mu(\text{SUSY})$ by increasing m_t . We checked that, if we use $m_t = 179.4$ GeV which is the 1- σ upper bound on the top quark mass [9], the curves move to the right; approximately, the same upper bound on $a_\mu(\text{SUSY})$ is obtained for the Higgs mass larger than about 2–3 GeV compared to the previous case ($m_t = 174.3$ GeV).

We can also compare our results with those with the CMSSM. To maximize $a_\mu(\text{SUSY})$ in the CMSSM framework, we repeat our analysis imposing $m_{10} = m_5 = m_{H5} = m_{\tilde{H}5}$. We found that the result for the CMSSM is independent of m_{max} as far as $m_{\text{max}} \geq 500$ GeV. The results are also shown in Fig. 2 in the dashed lines. As one can see, the maximum possible value for the CMSSM case is significantly smaller than that in the general SU(5) case since the number of the free parameters is much smaller. In particular, $\tan \beta \gtrsim 10$ is required in the CMSSM case to explain the E821 anomaly while $\tan \beta \gtrsim 7$ in the general SU(5) GUT approach for $m_{\text{max}} = 1$ TeV.

Now, we consider the second case with large $a_{\tilde{U}}$.⁵ In this case, the Higgs mass may be affected by the large trilinear coupling. To understand this fact, it is instructive to calculate the correction to the quartic coupling of the standard-model like Higgs boson which is approximately given by $H_{\text{SM}} \simeq H_u \sin \beta + H_d \cos \beta$. Denoting the potential of H_{SM} below the SUSY scale as $V = \frac{1}{2} \lambda (|H_{\text{SM}}|^2 - v^2)^2$, we obtain, at the tree level, $\lambda = \frac{1}{4} (g_2^2 + g_1^2) \cos^2 2\beta$ with g_2 and g_1 being the gauge coupling constants for SU(2)_L and U(1)_Y gauge interactions, respectively. If the trilinear coupling is large, however, the threshold correction to λ becomes sizable. Assuming a large hierarchy

⁵ If $a_{\tilde{U}}$ is large, color breaking minimum may exist and the origin of the squark potential may become a false vacuum. Such a situation is, however, cosmologically safe if the squark field is trapped in the false vacuum from in early universe. For example, thermal effect in the early universe can trap the squark field at the origin.

between the electroweak scale and the stop mass, and approximating $m_{\tilde{t}_1} \sim m_{\tilde{t}_2}$, the threshold correction to the quartic coupling from the stop loop is given by [13]

$$\Delta\lambda \simeq \frac{3}{8\pi^2} \left(\frac{y_t^2 A_{\tilde{t}}^2}{m_{\tilde{t}}^2} - \frac{1}{12} \frac{A_{\tilde{t}}^4}{m_{\tilde{t}}^4} \right) \sin^4 \beta. \quad (18)$$

Here $y_t = [Y_U]_{33}$ and $A_{\tilde{t}} = [A_{\tilde{U}}]_{33}$, and the fitting formula for $A_{\tilde{t}}$ is given by

$$A_{\tilde{t}}(\mu = \mu_{\text{SUSY}}) \simeq 1.70 M_{1/2} + 0.24 a_{\tilde{U}}, \quad (19)$$

where we used $\mu_{\text{SUSY}} = 500$ GeV and $\tan \beta = 5$. Notice that $\Delta\lambda$ stays finite even if $m_{\tilde{t}}$ increases as far as the ratio $A_{\tilde{t}}/m_{\tilde{t}}$ is fixed. When $a_{\tilde{U}} = 0$ (or $a_{\tilde{U}}$ is small), $A_{\tilde{t}}$ is small and hence the trilinear coupling does not affect the Higgs mass so much. If a large value of $a_{\tilde{U}}$ is adopted, however, $A_{\tilde{t}}$ is enhanced and $\Delta\lambda$ can be close to ~ 0.1 . In this case, m_h is drastically enhanced even if the stops are relatively light.

Of course, $a_{\tilde{U}}$ also affects other parameters. One important effect is that the μ_H parameter increases as $A_{\tilde{t}}$ increases. This is because the trilinear coupling changes the value of $m_{H_u}^2$ through the renormalization group effect. As a result, too large $a_{\tilde{U}}$ results in a suppressed value of $a_\mu(\text{SUSY})$. In addition, when $a_{\tilde{U}}$ is large, the squared masses of stops become negative unless m_{10} is large enough. In this case, large hierarchy between $m_{\tilde{5}}$ and m_{10} is necessary to maximize $a_\mu(\text{SUSY})$ since enhanced $a_\mu(\text{SUSY})$ requires small value of $m_{\tilde{5}}$. As a result, with large value of $a_{\tilde{U}}$, the SO(10) relation (i.e., $m_{\tilde{5}} = m_{10}$) is incompatible with the condition to maximize $a_\mu(\text{SUSY})$.

To study the effect of $a_{\tilde{U}}$, we vary $a_{\tilde{U}}$ as well as other soft parameters and obtain the maximum possible value of $a_\mu(\text{SUSY})$ as a function of m_h . In Fig. 3, we plot the maximum possible value as a function of $a_{\tilde{U}}$ for $m_h = 113.5$ GeV. For the $\tan \beta = 5$ case, by assuming a large value of $a_{\tilde{U}}$, the SUSY contribution to the muon MDM is significantly enhanced relative to the case of $a_{\tilde{U}} = 0$. In the general SU(5) case, we find a big increase of the upper bound on $a_\mu(\text{SUSY})$ at around $a_{\tilde{U}} \sim 300$ GeV. This can be understood as follows. When $a_{\tilde{U}}$ is large, the lightest Higgs mass can be enhanced by a large value of $\Delta\lambda$ without pushing up the sfermion masses. As a result, the slepton masses and the chargino masses may be small even for a large value of m_h , and hence the SUSY contribution to the muon MDM may become

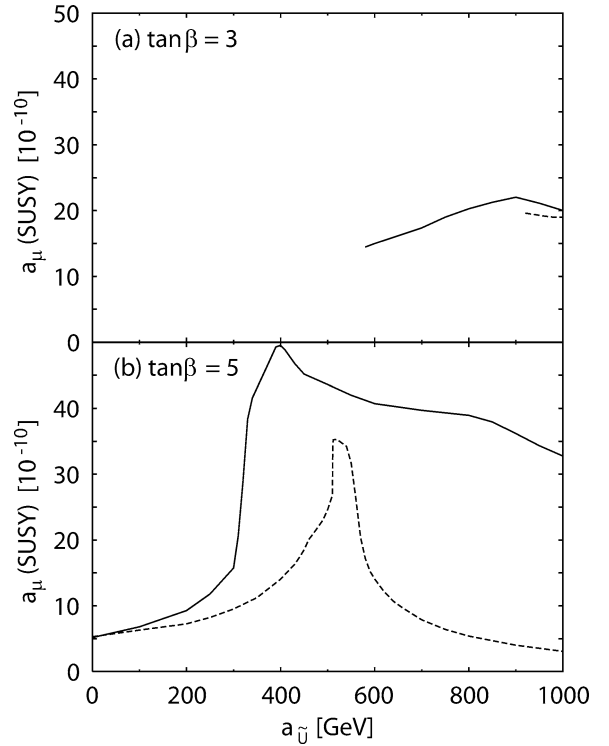


Fig. 3. Maximum possible value of $a_\mu(\text{SUSY})$ in units of 10^{-10} as a function of $a_{\tilde{U}}$ for $\tan \beta = 3$ and 5. The solid lines are for the general SU(5) case and the dashed lines are for the CMSSM case, and we adopt $m_{\text{max}} = 1$ TeV. The lightest Higgs mass m_h is fixed to be 113.5 GeV. Notice that, for $\tan \beta = 3$, $a_{\tilde{U}}$ has to be large enough to push up the Higgs mass, and hence the lower bound on $a_{\tilde{U}}$ exists.

large. We see that $a_\mu(\text{SUSY})$ can be as large as the deviation between $a_\mu(\text{E821})$ and $a_\mu(\text{SM})$ even with a small value of $\tan \beta$, like $\tan \beta = 5$. In addition, the trilinear coupling may also play a significant role in the case of the CMSSM, as can be seen in Fig. 3.

In summary, we have discussed the muon magnetic dipole moment and the Higgs mass in the framework of the supersymmetric SU(5) models. Importantly, we have *not* adopted the assumption of the universal scalar mass but have treated all the relevant parameters to be free. Then, we found that the maximum possible value of the SUSY contribution to the muon MDM becomes larger compared to the case of the universal scalar mass. When the trilinear coupling is small, to maximize $a_\mu(\text{SUSY})$ for a fixed value of m_h , soft SUSY breaking masses for the sfermions at the GUT scale should be small while those for the Higgses (as

well as the gaugino masses) are preferred to be finite. It is interesting that such a situation may be realized in, for example, the gaugino-mediated SUSY breaking scenario with the Higgs multiplets in the bulk [14]. In such a framework, the gauge and Higgs multiplets directly feel the effect of the SUSY breaking, and hence $M_{1/2}$, m_{H5} , and $m_{H\bar{5}}$ are finite while $m_{\bar{5}}$ and m_{10} vanish at the cutoff scale. In addition, it has been also shown that, if the trilinear scalar coupling for the stop is large, constraint from the lightest Higgs mass is drastically relaxed. In this case, the SUSY contribution to the muon MDM can completely explain the E821 anomaly even for $\tan\beta = 5$.

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